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Book Review

"*Asymptotic Perturbation Theory of Waves*", by Lev Ostrovsky, Imperial College Press, 2015; ISBN 978-1-84816-235-8

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There are many problems in natural sciences where the governing equations are a set of nonlinear evolution equations containing a lot of variables. There are two approaches for obtaining adequate approximation to the solutions of the problem, numerical method and the analytic approximations. If there is a small parameter present, the use of perturbation or asymptotic methods can allow for real progress to be made in trying to describe and understand the solution properties. "The game consists to find an approximate solution of a physical problem depending on a small parameter…" (P.-Y. Lagrée http://www.lmm.jussieu.fr/~lagree/COURS/M2MHP/MAE.pdf).

This book is devoted to asymptotic perturbation schemes for periodic and solitary waves. It is organized as follows.

Chapter 1, *Perturbed Oscillations and Waves*: *Introductory Examples*, describes the essence of perturbation theory and gives some simple illustrative examples of perturbation problems. Included are the three types of quasi-harmonic motions, namely, the linear oscillator with damping, the Duffing oscillator, and the Van der Pol oscillator. Two examples, based on the Klein–Gordon equation illustrate the distinction between linear and nonlinear waves. The important role of resonance is emphasized.

Chapter 2 concerns with *Perturbation Method for Quasi-Harmonic Waves*. The asymptotic scheme is studied here for a system of weakly nonlinear equations with dispersion. This approach is used in the analysis of a wide variety of complicated physical phenomena, including, among others, two-dimensional propagation of sound in a smoothly inhomogeneous medium, the propagation of a plane, transversely polarized electromagnetic wave in a model of a rarefied dielectric, and the resonance interaction of three waves.

Chapter 3 is devoted to *Perturbation Method for Non-Sinusoidal Waves*. The construction of a closed set of equations determining slow variation of the wave parameters is performed, in case of periodic but not harmonic basic solutions. This methodology is then applied to Lagrangian systems with lagrangian density containing slowly varying parameters. The Whitham's averaged variational principle is derived, as the first approximation of the perturbation approach to the Lagrangian systems. An analytical solution of one dimensional equation for the wave amplitude is obtained also, and discussed briefly.

Chapter 4 addresses basic issues of *Nonlinear Waves of Modulation*. The standard modulational approach is to envisage a weakly nonlinear monochromatic form for solutions, and to derive equations describing the evolution of its envelope. After a brief presentation of the method, two widely used nonlinear equations, Klein–Gordon (KG) and Korteweg–de Vries (KdV), are described and investigated. A thorough analysis of the KdV equation, based on an elegant Whitham' method is performed. The wave evolution from the initial condition in the form of a stepwise perturbation is considered.

Chapter 5 concentrates on *Perturbation Methods for Solitary Waves and Fronts*. The exposition is based on an analysis of representative objects taken from fluid mechanics, nonlinear optics and plasma

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physics. The algorithms considered refer to conservative solitary waves such as a soliton and a kink i.e. the class of localized waves which tend to constant at infinity. First, the modified, "quasi-stationary" perturbation scheme is described. Next, the more general, non-stationary approach is developed, which is applicable to solitons in systems close to exactly integrable. Then, the inverse scattering perturbation scheme for solitons is briefly elucidated. It is shown that the "direct" and "inverse" schemes satisfy the "equivalence principle"; they essentially lead to the same results.

Chapter 6 deals with *Perturbed Solitons*. It starts by applying the direct perturbation scheme to the KdV equation with a small additional term. Next, the KdV equation with dissipation is treated, and the radiation field magnitude is estimated. The similar techniques are applied to the nonlinear KG equation and the nonlinear Schrödinger equation (NSE). An important and interesting non-integrable fourth-order system, the rotational KdV equation, taking into account the effect of Earth's rotation on internal and surface gravity waves in the ocean, is analysed also. The chapter concludes by presenting the results of the geometrical aspects of solitons (refraction, transverse stability of a soliton, circular fronts, etc.).

Chapter 7 focuses on *Interaction and Ensembles of Solitons and Kinks*. Various aspects regarding the solitons interaction in a Lagrangian system are reviewed, and the "classical mechanics" of "rarefied" ensemble of solitons is formulated. Three types of soliton interactions: repulsion, attraction, and bound states are analysed. A self-similar solution of a generalized KdV equation is constructed and the non-integrable case of this equation is discussed. The next sections present the dynamics of soliton ensembles in the form of quasi-periodic chain of weakly interacting solitons, as well as various solitons and multi-solitons interaction in electromagnetic lines. Following this discussion, the Gardner equation, which is a generalization of the KdV equation containing both quadratic and cubic nonlinearities, is investigated. In the next section, a representative example is given to illustrate the computational effectiveness of the scheme presented above, applied to the description of a sequence of gravitational internal solitons observed on the ocean. At last, an interesting analysis of N-dimensional solitons ($N = 2, 3$), based on Swift-Hohenberg equation is performed.

Chapter 8 discusses the *Dissipative and Active Systems* and especially *Autowaves*, i.e., nonlinear spatio-temporal structures propagating through the medium. The Burgers equation is presented first. Then, the KdV-like equation is analysed which predicts the existence of autosolitons and illustrates the phenomenon of explosive instability. In sequel, the various aspects of soliton propagation on the background of a long wave are described. The chapter ends with two examples of reaction–diffusion systems, namely, the Kolmogorov–Petrovskii–Piskunov equation in the Fischer form, and the FitzHugh–Nagumo type equations.

The book under review is well written and to a certain extent is self-contained. The author of the book is the leading expert in the subject, and he made the effort to make the book accessible for everyone, from graduate students to applied mathematicians and physicists, working in this area. Some familiarity with the perturbation methods is not necessary, but desirable. However, the perceptible dissonance between the typography of the text and the math output does not help the legibility of the book.